

On the Progression of Knowledge in the Situation Calculus

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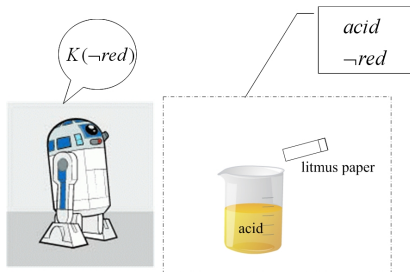
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Litmus test example [Moore 1985]

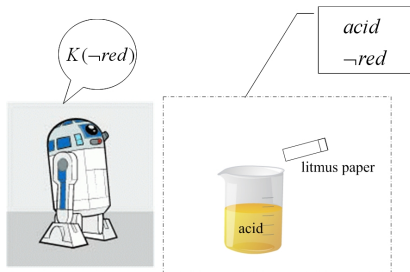


The initial KB:

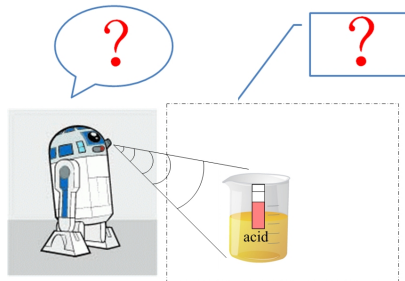
$\{acid, \neg red, K(\neg red)\}$



Litmus test example [Moore 1985]



The initial KB:
 $\{acid, \neg red, K(\neg red)\}$



The new KB:
 $\{acid, red, K(acid), K(red)\}$



Main problems

How to update the knowledge base (KB) after an action is executed?

 The KB includes both objective and subjective knowledge

- 1 Whether the new KB is definable in first-order modal logic?
- 2 Whether the new KB is finite?
- 3 Whether the new KB is computable?



Our main works

- Definition of knowledge progression
- Progression wrt. physical actions
 - this reduces to forgetting in first-order modal logic
 - progression wrt local-effect physical actions
- Progression wrt. sensing actions



Situation calculus and basic action theories

- Actions, e.g., $move(x, y)$; Fluents, e.g., $on(x, y, s)$
- Situations, S_0 , $do(a, s)$
- A basic action theory (BAT) has the form

$$\mathcal{D} = \Sigma \cup \mathcal{D}_{una} \cup \mathcal{D}_{ap} \cup \mathcal{D}_{ss} \cup \mathcal{D}_{S_0}, \text{ where}$$

- $\mathcal{D}_{ap}$: action precondition axioms
- $\mathcal{D}_{ss}$: successor state axioms, e.g.,

$$\begin{aligned} red(do(a, s)) &\equiv a = test \wedge acid(s) \vee red(s), \\ acid(do(a, s)) &\equiv acid(s) \wedge a \neq dilute \end{aligned}$$

- $\mathcal{D}_{S_0}$: the initial KB, a finite set of sentences about S_0 , e.g.

$$acid(S_0), K(\neg red, S_0)$$



Progression

An intuitive definition

Let α be a ground action, and let S_α denote $do(\alpha, S_0)$.

A progression of $\mathcal{D}_{S_0}$ wrt α is a set of sentences $\mathcal{D}_{S_\alpha}$ s.t.

- 1 Just as $\mathcal{D}_{S_0}$ is about S_0 only, $\mathcal{D}_{S_\alpha}$ should be about S_α only;
- 2 the old theory $\mathcal{D}$ and the new theory $\mathcal{D}' = (\mathcal{D} - \mathcal{D}_{S_0}) \cup \mathcal{D}_{S_\alpha}$ should be equivalent wrt the possible futures of S_α .



Progression

A formal definition

$\mathcal{D}_{S_\alpha}$ is a progression of $\mathcal{D}_{S_0}$ wrt α , written $\mathcal{D}_{S_0} \xRightarrow{\alpha} \mathcal{D}_{S_\alpha}$, if

- 1 $\mathcal{D}_{S_\alpha}$ is about S_α only, $\mathcal{D} \models \mathcal{D}_{S_\alpha}$, and
- 2 for every $M \models \mathcal{D}'$, there is $M' \models \mathcal{D}$ such that $M \sim_{S_\alpha} M'$.

How to define $M \sim_{S_\alpha} M'$ in the presence of knowledge?

- We adapt the concept of bisimulation from modal logic



Bisimulation

Definition

$M \sim_{S_\alpha} M'$ if there is a relation B between their situations s.t. whenever $\delta B \delta'$,

- 1 δ and δ' agree on all ordinary fluents;
- 2 The forth condition: for all ρ accessible from δ , there is ρ' accessible from δ' s.t. $\rho B \rho'$;
- 3 The back condition: for all ρ' accessible from δ' , there is ρ accessible from δ s.t. $\rho B \rho'$.



Progression wrt physical actions

Theorem (1)

Let $\mathcal{D}_{S_0}$ be $\phi[S_0]$ where ϕ is in FO modal logic, and α a physical action. Let

$$kforget(\phi \wedge K\mathcal{D}_{ss}^*[\alpha, S_0], \vec{F}) \Leftrightarrow \psi,$$

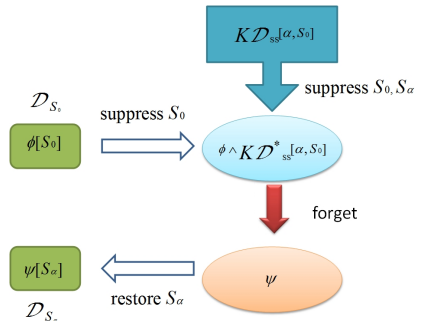
where ψ is in second-order modal logic. Then $\mathcal{D}_{S_0} \stackrel{\alpha}{\Rightarrow} \psi[S_\alpha]$.

👉 So for physical actions, progression of knowledge reduces to forgetting predicates in first order modal logic.



Progression wrt physical actions

- 1 Suppress situation argument of the initial KB and those of the knowledge on SSAs.
- 2 Forget the predicates describing the initial situation.
- 3 Restore the new situation argument to the result of knowledge forgetting.



Progression wrt local-effect physical actions

Theorem (2)

*Let $\mathcal{D}$ be local-effect, and α a physical action. Let $\mathcal{D}_{S_0}$ be $\phi[S_0]$ where ϕ is in $\exists$ -DNF. Then there is ψ in FO modal logic s.t.
 $kforget(\phi \wedge K\mathcal{D}_{ss}^*[\Omega], \Omega) \Leftrightarrow \psi$, and $\mathcal{D}_{S_0} \xRightarrow{\alpha} \psi(\vec{F}/\vec{F}')[S_\alpha]$.*

 In this case, progression of knowledge is definable in first-order modal logic.



Local-effect physical actions

- Local-effect actions: if an action $A(\vec{c})$ changes the truth value of an atom $F(\vec{a}, s)$, then $\vec{a}$ is contained in $\vec{c}$.
- Let $\Omega = \{F(\vec{a}) \mid \vec{a} \text{ is contained in } \vec{c}\}$. Then the predicates $F(\vec{x})$ (i.e. $F(\vec{x}, s)$ at S_0) and $F'(\vec{x})$ (i.e. $F(\vec{x}, s)$ at S_α) are different at only instances from Ω .
- Then forgetting predicate F can be obtained from forgetting atoms in Ω and then replacing F by F' in the result.

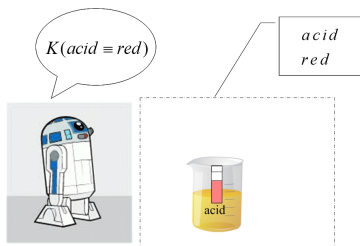


$\exists$ -DNF

- ϕ is in $\exists$ -DNF if it is of the form $\exists \vec{x}(\phi_1 \vee \dots \vee \phi_n)$, where each ϕ_i is of the form $\alpha \wedge K\beta \wedge \bigwedge_i \forall \vec{x}_i M\gamma_i$, where α, β , and γ_i 's are all objective, and $M\gamma$ abbreviates for $\neg K\neg\gamma$
- $\exists$ -DNF goes beyond formulas without quantifying-in
- Let ϕ be in $\exists$ -DNF. Then the result of forgetting an atom in ϕ is definable in first-order modal logic.



Example: Progression wrt "test"



- $\mathcal{D}_{S_0} = \{\text{acid}(S_0), K(\neg \text{red}, S_0)\}.$
- $S_1 = \text{do}(\text{test}, S_0).$
- $\mathcal{D}_{S_1}$ includes:
 $\text{red}(\text{do}(a, s)) \equiv a = \text{test} \wedge \text{acid}(s) \vee \text{red}(s),$
 $\text{acid}(\text{do}(a, s)) \equiv \text{acid}(s) \wedge a \neq \text{dilute}.$
- $k\text{forget}(\phi, \text{red}),$ where
 $\phi = \text{acid} \wedge K\neg \text{red} \wedge K(\text{red}' \equiv \text{acid} \vee \text{red})$
- $\mathcal{D}_{S_1} =$
 $\{\text{acid}(S_1), \text{red}(S_1), K(\text{acid} \equiv \text{red}, S_1)\}$



Progression wrt sensing actions

Theorem (3)

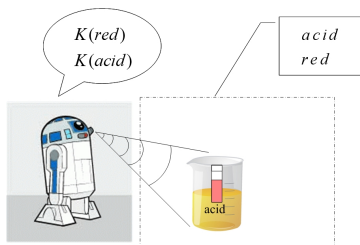
Assume that $\mathcal{D}_{S_0}$ does not contain negative knowledge.
 Let α be $\text{sense}_\psi(\vec{c})$, where $\psi(\vec{x})$ is an objective formula. Then
 $\mathcal{D}_{S_0} \xrightarrow{\alpha} \mathcal{D}_{S_0}(S_0/S_\alpha) \cup \{K(\psi(\vec{c}), S_\alpha) \vee K(\neg\psi(\vec{c}), S_\alpha)\}.$

👉 negative knowledge: $\neg K(\phi).$

👉 positive knowledge: $K(\phi), K(\neg\phi).$



Example: Progression wrt “sense”



- $\mathcal{D}_{S_1} = \{acid(S_1), red(S_1), K(acid \equiv red, S_1)\}$
- $S_2 = do(sense_{red}, S_1)$
- $\{acid(S_2), red(S_2), K(red \equiv acid, S_2), K(red, S_2) \vee K(\neg red, S_2)\}$
- $\mathcal{D}_{S_2} = \{K(red, S_2), K(acid, S_2)\}$



Conclusions

- Adapted the concept of bisimulation and extended Lin & Reiter's notion of progression to accommodate knowledge
- Showed for physical actions, progression of knowledge reduces to forgetting predicates in FO modal logic
- Showed for local-effect physical actions, when the initial KB is in $\exists$ -DNF, progression of knowledge is definable in FO modal logic
- Showed for sensing actions, when the initial KB does not contain negative knowledge, progression can be simply achieved
- Extended our results to the multi-agent case (see the paper)

